

Heavy Hybrids: decay to and mixing with Heavy Quarkonium

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Abstract. We report on a recent QCD based research on hybrid mesons containing $c\bar{c}$ or $b\bar{b}$ quarks. We present results for the spectrum, the decay widths to heavy quarkonium, and the role of mixing with the latter. We point out that mixing with heavy quarkonium provides a potentially large source of spin symmetry breaking. We identify candidates to hybrid mesons among the so called XYZ states in the charmonium and bottomonium spectrum.

1 Introduction

The so called XYZ states in the charmonium and bottomonium spectrum do not fit in the usual potential model expectations (see [1] for a recent review). A number of models have been proposed to understand them, ranging from compact tetraquark states to just kinematical enhancements caused by the heavy-light meson pair thresholds. We pursue here a QCD based approach based on the fact that charm and bottom masses are much larger than the typical QCD scale Λ_{QCD} , and hence Non-Relativistic QCD (NRQCD) [2] holds for these states. Furthermore, if we focus on a region of the spectrum much smaller than Λ_{QCD} , we should be able to build an effective theory in that region, by integrating out Λ_{QCD} , in a way similar to the strong coupling regime of Potential NRQCD (pNRQCD) [3]. The static limit is relevant for such a construction and the spectrum in that limit is known from lattice QCD in the case of $n_f = 0$ (no light quarks) [4]. In the Born-Oppenheimer (BO) approximation, each energy level in the static case plays the role of a potential in a Schrödinger equation for the dynamical states build on that static energy level [5]. The static spectrum is displayed in fig. 1.

The ground state corresponds to the potential for heavy quarkonium states, namely the one that it is usually input in potential models. The higher levels correspond to gluonic excitations and are called hybrid potentials. If we are interested in states of a certain energy, we must in principle take into account all the potentials below that energy, since the states build on different potentials may influence each other through $1/m_Q$ corrections, m_Q being the mass of the heavy quarks ($Q = c, b$). We shall focus here on the lower lying hybrid states build out of Π_u and Σ_u^- , and will address the question on how they talk to states build out of Σ_g^+ (heavy quarkonium). The Σ_g^+ states far below the energy of the Π_u and Σ_u^- can be integrated out and may contribute to the decay width, whereas the Σ_g^+ states in

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2 Spectrum

In the BO approximation, the calculation of the heavy hybrid spectrum reduces to solving the Schrödinger equation with a potential $V = V(r, \Lambda_{QCD})$ that has a minimum in $r = r_0 \sim 1/\Lambda_{QCD}$ ($r = |\mathbf{r}|$, \mathbf{r} being the relative coordinate). Hence the energy of the small fluctuation about that minimum is $E \sim \sqrt{\Lambda_{QCD}^3/m_Q} \ll \Lambda_{QCD} \ll m_Q$. Consequently, we are in a situation analogous to the strong coupling regime of pNRQCD in which the scale Λ_{QCD} may be integrated out. It then makes sense to restrict the study to the lowest lying heavy hybrid potentials, Σ_u^- and Π_u , since the gap to the next state is parametrically $O(\Lambda_{QCD})$.

The potentials associated with Σ_u^- and Π_u are degenerated at short distances (see fig. 1). In pNRQCD this is easily understood as they correspond to different projections with respect to \mathbf{r} of the same operator $tr(\mathbf{B}(\mathbf{0}, t)\mathbf{O}(\mathbf{0}, \mathbf{r}, t))$ (we have set the center of mass coordinate $\mathbf{R}=\mathbf{0}$, see [3] for the notation). It is then natural to associate to the lower lying heavy hybrids a vectorial wave function $\mathbf{H}(\mathbf{0}, \mathbf{r}, t)$ with the same symmetry properties as the above operator, such that its projection to \mathbf{r} evolves with $V_{\Sigma_u^-}$ and its projection orthogonal to \mathbf{r} with V_{Π_u} . We then have the following Lagrangian density,

$$\mathcal{L} = \text{tr} \left(H^{i\dagger} \left(\delta_{ij} i \partial_0 - h_{Hij} \right) H^j \right) \quad , \quad h_{Hij} = \left(-\frac{\nabla^2}{m_Q} + V_{\Sigma_u^-}(r) \right) \delta_{ij} + \left(\delta_{ij} - \hat{r}_i \hat{r}_j \right) \left[V_{\Pi_u}(r) - V_{\Sigma_u^-}(r) \right] \quad , \quad (1)$$

$\hat{\mathbf{r}} = \mathbf{r}/|\mathbf{r}|$. $\mathbf{H} = \mathbf{H}(\mathbf{0}, \mathbf{r}, t)$ is a matrix in spin space and the trace is over spin indices. h_{Hij} above does not depend on the spin of the quarks, and hence it is invariant under spin symmetry transformations, but it does depend on the total angular momentum of the gluonic degrees of freedom \mathbf{L}_g , in this case $L_g = 1$ as it is apparent from the vectorial character of \mathbf{H} . The P and C associated to a Hybrid state with quark-antiquark orbital angular momentum L and quark-antiquark spin S become,

$$P = (-1)^{L+1} \quad , \quad C = (-1)^{L+S+1} \quad . \quad (2)$$

Leaving aside the spin of the quarks, it is convenient to express \mathbf{H} in a basis of eigenfunctions of $\mathbf{J} = \mathbf{L} + \mathbf{L}_g$, where \mathbf{L} is the orbital angular momentum of the quarks. This is achieved using Vector Spherical Harmonics [11],

$$\mathbf{H}(\mathbf{0}, \mathbf{r}, t) = \frac{e^{-iEt}}{r} \left(P_0^+(r) \mathbf{Y}_{00}^{L=1}(\theta, \phi) + \sum_{J=1}^{\infty} \sum_{M=-J}^J \left[P_J^+(r) \mathbf{Y}_{JM}^{L=J+1}(\theta, \phi) + P_J^0(r) \mathbf{Y}_{JM}^{L=J}(\theta, \phi) + P_J^-(r) \mathbf{Y}_{JM}^{L=|J-1|}(\theta, \phi) \right] \right) \quad . \quad (3)$$

Note that \mathbf{J} is a conserved quantity thanks to heavy quark spin symmetry. The eigenvalue problem then reduces for $J \neq 0$ to

$$\left[-\frac{1}{m_Q} \frac{\partial^2}{\partial r^2} + \begin{pmatrix} \frac{(J-1)J}{m_Q r^2} & 0 \\ 0 & \frac{(J+1)(J+2)}{m_Q r^2} \end{pmatrix} + V_{\Sigma_u^-}(r) + V_q(r) \begin{pmatrix} \frac{J+1}{2J+1} & \frac{\sqrt{(J+1)J}}{2J+1} \\ \frac{\sqrt{(J+1)J}}{2J+1} & \frac{J}{2J+1} \end{pmatrix} \right] \begin{pmatrix} P_J^-(r) \\ P_J^+(r) \end{pmatrix} = E \begin{pmatrix} P_J^-(r) \\ P_J^+(r) \end{pmatrix}$$

$$\left(-\frac{1}{m_Q} \frac{\partial^2}{\partial r^2} + \frac{J(J+1)}{m_Q r^2} + V_{\Pi_u}(r) \right) P_J^0(r) = E P_J^0(r) \quad , \quad (4)$$

where $V_q(r) = V_{\Pi_u}(r) - V_{\Sigma_u^-}(r)$. For $J = 0$ only the equation for $P_J^+(r)$ remains.

We are going to approximate $V_{\Pi_u}(r)$ and $V_{\Sigma_u^-}(r)$ by simple functions that have the correct asymptotic behavior at short and long distances, and fit well the lattice data of ref. [4]. For $V_{\Sigma_u^-}(r)$ it is enough to take a Cornell-like potential in order to get a good description of lattice data. We impose

the linear term to be equal to the one for heavy quarkonium ($V_{\Sigma_c^+}(r)$) as dictated by the QCD effective string theory (EST) [12], and the Coulomb term to be related to the one of $V_{\Sigma_c^+}(r)$ as dictated by perturbation theory. Hence, only an additive constant is left as a free parameter, which can in turn be related to the corresponding constant for $V_{\Sigma_c^+}(r)$ through the lattice data. The last constant is fixed by fitting the heavy quarkonium spectrum. For $V_{\Pi_u}(r)$ a Cornell-like form does not fit lattice data well at intermediate distances. Hence, we take a slightly more complicated function for it, we substitute the Coulomb term by a rational function with five free parameters. At short and long distances we impose $V_{\Pi_u}(r) - V_{\Sigma_c^+}(r)$ to be compatible with the weak coupling pNRQCD results at NLO [3] and with the EST results at NLO [12] respectively. The remaining two free parameters are fitted to lattice data.

Using the potentials above as an input we solve numerically (4) and obtain the results displayed in tables 1 and 2. We also display the results for heavy quarkonium, obtained with a Cornell potential that fits well the lattice data for Σ_c^+ of [4].

In table 3 we show possible identifications with XYZ states. According to this table only spin zero hybrids would have been observed. It is interesting to notice that $Y(4008)$, $Y(4360)$ and $Y(4660)$ would correspond to the ground state and the lower excitations of the $(s/d)_1$ state. However, the three states have been observed to decay to vector quarkonium, which violates spin symmetry [13]. This is also so for $Y_b(10890)$. In fact, from the 1^{--} candidates only for $X(4630)$ no spin symmetry violating decay has been observed. This motivates the study of mixing with heavy quarkonium in section 4. We report in the next section on our results for the decay widths to lower lying heavy quarkonium states.

3 Decay

Since we are interested in the lowest lying heavy hybrid states, it is enough for us to consider an effective theory for energy fluctuations $E \ll \Lambda_{QCD}$ around those states. The energy gap to the lower lying heavy quarkonium states is greater than Λ_{QCD} . Hence the lower lying heavy quarkonium states can be integrated out, which will give rise to an imaginary potential ΔV for the heavy hybrid states, which in turn will produce a decay width for them, $\Gamma_{H \rightarrow S} = -2 \langle H | \text{Im} \Delta V | H \rangle$. This is much in the same way as integrating out hard gluons produces operators with imaginary matching coefficients in NRQCD [14]. Furthermore, if we assume that the energy gap ΔE fulfills $\Delta E \gg \Lambda_{QCD}$, and that the process is dominated by short distances, the integration can be done using the weak coupling regime of pNRQCD [3, 15]

We obtain,

$$\text{Im} \Delta V = -\frac{2}{3} \frac{\alpha_s T_F}{N_c} \sum_n r^j |S_n\rangle \langle S_n| r^j (i\partial_t - E_n)^3, \quad (5)$$

$T_F = 1/2$, $N_c = 3$, and α_s is the QCD strong coupling constant. E_n is the energy of the n -th heavy quarkonium state, S_n . The calculation is reliable only for those states that fulfill $\Delta E_n \gg \Lambda_{QCD}$, for which we identify,

$$\Gamma(H_m \rightarrow S_n) = \frac{4\alpha_s T_F}{3 N_c} \langle H_m | r^j | S_n \rangle \langle S_n | r^j | H_m \rangle (\Delta E_{mn})^3, \quad (6)$$

where m stands for NL_J , the quantum numbers of the heavy hybrid (H_m), n for $N'L'$, the quantum numbers of the heavy quarkonium (S_n), and ΔE_{mn} is the energy difference between them. For consistency, $\langle S_n | r^j | H_m \rangle \Delta E_{mn}$ should also be small, otherwise the multipole expansion built in weak coupling pNRQCD would not be justified. The structure of the decay width above implies that no heavy hybrid with $L = J$ decays to heavy quarkonium at this order. This selects $X(4160)$ as the preferred candidate for the $1p_1$ state in table 3 since no decay to charmonium has been observed, as opposite to $X(4140)$. The numerical values of the decay widths are given in table 4. The scale of α_s is taken at $\mu = \Delta E_{mn}$. At this order, the decays respect heavy quark spin symmetry, and hence the spin of the heavy quarks

NL_J	w-f	$c\bar{c}$	Hybrid	$S = 0$ \mathcal{J}^{PC}	$S = 1$ \mathcal{J}^{PC}	Λ_η^ϵ	
1s	S	3068	4486	0^{-+}	1^{--}	Σ_g^+	
2s	S	3678		0^{-+}	1^{--}	Σ_g^+	
3s	S	4131		0^{-+}	1^{--}	Σ_g^+	
1p ₀	H ⁺	4512		0^{++}	1^{+-}	Σ_u^-	
4s	S			0^{-+}	1^{--}	Σ_g^+	
2p ₀	H ⁺			4920	0^{++}	1^{+-}	Σ_u^-
3p ₀	H ⁺			5299	0^{++}	1^{+-}	Σ_u^-
4p ₀	H ⁺			5642	0^{++}	1^{+-}	Σ_u^-
1p	S	3494	4011	1^{+-}	$(0, 1, 2)^{++}$	Σ_g^+	
2p	S	3968		1^{+-}	$(0, 1, 2)^{++}$	Σ_g^+	
1(s/d) ₁	H ⁺⁻	4369		1^{--}	$(0, 1, 2)^{-+}$	$\Pi_u \Sigma_u^-$	
1p ₁	H ⁰			4145	1^{++}	$(0, 1, 2)^{+-}$	Π_u
2(s/d) ₁	H ⁺⁻			4355	1^{--}	$(0, 1, 2)^{-+}$	$\Pi_u \Sigma_u^-$
3p	S			1^{+-}	$(0, 1, 2)^{++}$	Σ_g^+	
2p ₁	H ⁰			4511	1^{++}	$(0, 1, 2)^{+-}$	Π_u
3(s/d) ₁	H ⁺⁻			4692	1^{--}	$(0, 1, 2)^{-+}$	$\Pi_u \Sigma_u^-$
4(s/d) ₁	H ⁺⁻			4718	1^{--}	$(0, 1, 2)^{-+}$	$\Pi_u \Sigma_u^-$
4p	S			4727	1^{+-}	$(0, 1, 2)^{++}$	Σ_g^+
3p ₁	H ⁰	4863		1^{++}	$(0, 1, 2)^{+-}$	Π_u	
5(s/d) ₁	H ⁺⁻	5043		1^{--}	$(0, 1, 2)^{-+}$	$\Pi_u \Sigma_u^-$	
5p	S	5055		1^{+-}	$(0, 1, 2)^{++}$	Σ_g^+	
1d	S	3793		4231	2^{-+}	$(1, 2, 3)^{--}$	Σ_g^+
2d	S	4210			2^{-+}	$(1, 2, 3)^{--}$	Σ_g^+
1(p/f) ₂	H ⁺⁻	4579			2^{++}	$(1, 2, 3)^{+-}$	$\Pi_u \Sigma_u^-$
1d ₂	H ⁰		4334		2^{--}	$(1, 2, 3)^{-+}$	Π_u
2(p/f) ₂	H ⁺⁻		4563		2^{++}	$(1, 2, 3)^{+-}$	$\Pi_u \Sigma_u^-$
3d	S		2^{-+}		$(1, 2, 3)^{--}$	Σ_g^+	
2d ₂	H ⁰		4693		2^{--}	$(1, 2, 3)^{-+}$	Π_u
3(p/f) ₂	H ⁺⁻		4886		2^{++}	$(1, 2, 3)^{+-}$	$\Pi_u \Sigma_u^-$
4d	S		4916		2^{-+}	$(1, 2, 3)^{--}$	Σ_g^+
4(p/f) ₂	H ⁺⁻		4923		2^{++}	$(1, 2, 3)^{+-}$	$\Pi_u \Sigma_u^-$
3d ₂	H ⁰	5036	2^{--}		$(1, 2, 3)^{-+}$	Π_u	

Table 1. Charmonium (S) and Hybrid charmonium (H^{+-0}) energy spectrum computed with $m_c = 1.47 GeV$.

Masses are in MeV. States which only differ by the heavy quark spin ($S = 0, 1$) are degenerated. N is the principal quantum number, L the orbital angular momentum of the heavy quarks, J is L plus the total angular momentum of the gluons, S the spin of the heavy quarks and \mathcal{J} is the total angular momentum. For quarkonium, J coincides with L and it is not displayed. The last column shows the relevant potentials for each state. The $(s/d)_1$, p_1 and p_0 states are usually named H_1 , H_2 and H_3 respectively.

NL_J	w-f	$b\bar{b}$	Hybrid	$S = 0$ \mathcal{J}^{PC}	$S = 1$ \mathcal{J}^{PC}	Λ_η^ϵ
$1s$	S	9442		0^{-+}	1^{--}	Σ_g^+
$2s$	S	10009		0^{-+}	1^{--}	Σ_g^+
$3s$	S	10356		0^{-+}	1^{--}	Σ_g^+
$4s$	S	10638		0^{-+}	1^{--}	Σ_g^+
$1p_0$	H ⁺		11011	0^{++}	1^{+-}	Σ_u^-
$2p_0$	H ⁺		11299	0^{++}	1^{+-}	Σ_u^-
$3p_0$	H ⁺		11551	0^{++}	1^{+-}	Σ_u^-
$4p_0$	H ⁺		11779	0^{++}	1^{+-}	Σ_u^-
$1p$	S	9908		1^{+-}	$(0, 1, 2)^{++}$	Σ_g^+
$2p$	S	10265		1^{+-}	$(0, 1, 2)^{++}$	Σ_g^+
$3p$	S	10553		1^{+-}	$(0, 1, 2)^{++}$	Σ_g^+
$1(s/d)_1$	H ⁺⁻		10690	1^{--}	$(0, 1, 2)^{-+}$	$\Pi_u \Sigma_u^-$
$1p_1$	H ⁰		10761	1^{++}	$(0, 1, 2)^{-+}$	Π_u
$4p$	S	10806		1^{+-}	$(0, 1, 2)^{++}$	Σ_g^+
$2(s/d)_1$	H ⁺⁻		10885	1^{--}	$(0, 1, 2)^{-+}$	$\Pi_u \Sigma_u^-$
$2p_1$	H ⁰		10970	1^{++}	$(0, 1, 2)^{-+}$	Π_u
$5p$	S	11035		1^{+-}	$(0, 1, 2)^{++}$	Σ_g^+
$3(s/d)_1$	H ⁺⁻		11084	1^{--}	$(0, 1, 2)^{-+}$	$\Pi_u \Sigma_u^-$
$4(s/d)_1$	H ⁺⁻		11156	1^{--}	$(0, 1, 2)^{-+}$	$\Pi_u \Sigma_u^-$
$3p_1$	H ⁰		11175	1^{++}	$(0, 1, 2)^{-+}$	Π_u
$6p$	S	11247		1^{+-}	$(0, 1, 2)^{++}$	Σ_g^+
$5(s/d)_1$	H ⁺⁻		11284	1^{--}	$(0, 1, 2)^{-+}$	$\Pi_u \Sigma_u^-$
$1d$	S	10155		2^{-+}	$(1, 2, 3)^{--}$	Σ_g^+
$2d$	S	10454		2^{-+}	$(1, 2, 3)^{--}$	Σ_g^+
$3d$	S	10712		2^{-+}	$(1, 2, 3)^{--}$	Σ_g^+
$1(p/f)_2$	H ⁺⁻		10819	2^{++}	$(1, 2, 3)^{-+}$	$\Pi_u \Sigma_u^-$
$1d_2$	H ⁰		10870	2^{--}	$(1, 2, 3)^{-+}$	Π_u
$4d$	S	10947		2^{-+}	$(1, 2, 3)^{--}$	Σ_g^+
$2(p/f)_2$	H ⁺⁻		11005	2^{++}	$(1, 2, 3)^{-+}$	$\Pi_u \Sigma_u^-$
$2d_2$	H ⁰		11074	2^{--}	$(1, 2, 3)^{-+}$	Π_u
$5d$	S	11163		2^{-+}	$(1, 2, 3)^{--}$	Σ_g^+
$3(p/f)_2$	H ⁺⁻		11197	2^{++}	$(1, 2, 3)^{-+}$	$\Pi_u \Sigma_u^-$
$3d_2$	H ⁰		11275	2^{--}	$(1, 2, 3)^{-+}$	Π_u
$4(p/f)_2$	H ⁺⁻		11291	2^{++}	$(1, 2, 3)^{-+}$	$\Pi_u \Sigma_u^-$

Table 2. Bottomonium (S) and hybrid bottomonium (H⁺⁻⁰) energy spectrum computed with $m_b = 4.88\text{GeV}$.

Masses are in MeV. States which only differ by the heavy quark spin ($S = 0, 1$) are degenerated. N is the principal quantum number, L the orbital angular momentum of the heavy quarks, J is L plus the total angular momentum of the gluons, S the spin of the heavy quarks and \mathcal{J} is the total angular momentum. For quarkonium, J coincides with L and it is not displayed. The last column shows the relevant potentials for each state. The $(s/d)_1$, p_1 and p_0 states are usually named H_1 , H_2 and H_3 respectively.

State	M	J^{PC}	XYZ	M_{exp}	Γ_{exp}	J_{exp}^{PC}
$1(s/d)_1$	4011	$1^{--}, (0, 1, 2)^{+-}$	Y(4008)	4008^{+121}_{-49}	226 ± 97	1^{--}
$1p_1$	4145	$1^{++}, (0, 1, 2)^{+-}$	Y(4140)	4144.5 ± 2.6	15^{+11}_{-7}	$?^{?+}$
			X(4160)	4156^{+29}_{-25}	139^{+113}_{-65}	$?^{?+}$
$2(s/d)_1$	4355	$1^{--}, (0, 1, 2)^{+-}$	Y(4360)	4361 ± 13	74 ± 18	1^{--}
$3(s/d)_1$	4692	$1^{--}, (0, 1, 2)^{+-}$	Y(4660)	4664 ± 12	48 ± 15	1^{--}
			X(4630)	4634^{+9}_{-11}	92^{+41}_{-32}	1^{--}
$2(s/d)_1$	10885	$1^{--}, (0, 1, 2)^{+-}$	$Y_b(10890)$	10888.4 ± 3	$30.7^{+8.9}_{-7.7}$	1^{--}

Table 3. Possible identifications of XYZ states in our spectrum. Masses and decay widths are given in MeV.

must be the same in the initial heavy hybrid state and in the final heavy quarkonium state. For the states in table 3, we are only able to put forward a couple of reliable estimates

$$\Gamma(Y(4360) \rightarrow h_c + X) \sim 19 \text{ MeV} \quad , \quad \Gamma(Y_b(10890) \rightarrow h_b + X) \sim 9.6 \text{ MeV} . \quad (7)$$

Notice that both numbers above are smaller than the measured total decay widths, as they should.

$NL_J \rightarrow N'L'$	ΔE (MeV)	$\langle r \rangle_{NL_J, N'L'} \text{ (GeV}^{-1}\text{)}$	$ \Delta E \langle r \rangle_{NL_J, N'L'} $	Γ (MeV)
$1p_0 \rightarrow 2s$	808	0.40	0.32	6.1
$2(s/d)_1 \rightarrow 1p$	861	0.63	0.54	19
$1p_0 \rightarrow 1s$	1569	-0.416	0.65	31
$1p_0 \rightarrow 2s$	1002	0.432	0.43	8.7
$2p_0 \rightarrow 2s$	1290	-0.137	0.18	1.9
$2p_0 \rightarrow 3s$	943	0.462	0.44	8.3
$2(s/d)_1 \rightarrow 1p$	977	0.470	0.46	9.6

Table 4. Decay widths for hybrid charmonium (above) and bottomonium (below) to lower lying charmonia and bottomonia respectively. We only display results for which $\Delta E \gtrsim 800 \text{ MeV}$ and $|\Delta E \langle r \rangle_{NL_J, N'L'}| \lesssim 0.6$.

4 Mixing

We shall focus here on the mixing with heavy quarkonium, basically because it is amenable to a systematic treatment. The mixing of spin zero heavy hybrids with spin one heavy quarkonium is relevant because it may explain the spin symmetry violating decays observed in some hybrid candidates. In the static limit, heavy quarkonium and heavy hybrids do not mix by construction (they are built as orthogonal states). Hence, the mixing must be due to $1/m_Q$ corrections to the BO approximation. These corrections may become large if there are energy levels of heavy quarkonium close to those of heavy hybrids. A way to systematically compute $1/m_Q$ corrections for heavy quarkonium was established in [16, 17] for the strong coupling regime of pNRQCD. The formalism in [16] can also be used to calculate the mixing potentials as we sketch below. We may generally consider an effective theory for energy fluctuations E around a hybrid state, such that $E \ll \Lambda_{QCD}$. If there is a heavy quarkonium

state close to that energy, we may expect it to modify the value of the energy E . This effective theory reads,

$$\mathcal{L}_{H+S} = \text{tr} \left(S^\dagger [i\partial_0 - h_s] S \right) + \text{tr} \left(H^{i\dagger} [i\delta_{ij}\partial_0 - h_{Hij}] H^j \right) + \text{tr} \left(S^\dagger V_S^{ij} \{ \sigma^i, H^j \} + \text{H.c.} \right). \quad (8)$$

The traces are over spin indices and

$$V_S^{ij} = V_S^{ij}(\mathbf{r}) = \delta^{ij} V_S^\Pi(r) + \hat{r}^i \hat{r}^j (V_S^\Sigma(r) - V_S^\Pi(r)) \quad (9)$$

is the mixing potential, $h_s = -\frac{\nabla^2}{m_Q} + V_{\Sigma_g^+}(r)$ and h_{Hij} is made explicit in (1). Notice that the last term in (8) mixes spin zero (one) heavy hybrids with spin one (zero) heavy quarkonium.

By matching NRQCD with the effective theory above at order $1/m_Q$ we obtain,

$$\frac{\frac{c_F}{2m_Q} \int_{-\frac{T}{2}}^{\frac{T}{2}} dt \langle \hat{\mathbf{r}} \mathbf{g} \mathbf{B}(\frac{\mathbf{r}}{2}, t) \hat{\mathbf{r}} \mathbf{g} \mathbf{B}(\mathbf{0}, -\frac{T}{2}) \rangle_\square}{\langle 1 \rangle_\square^{1/2} \langle \hat{\mathbf{r}} \mathbf{g} \mathbf{B}(\mathbf{0}, \frac{T}{2}) \hat{\mathbf{r}} \mathbf{g} \mathbf{B}(\mathbf{0}, -\frac{T}{2}) \rangle_\square^{1/2}} = 2V_S^\Sigma \frac{\sin\left(\frac{(V_{\Sigma_u^-} - V_{\Sigma_g^+})T}{2}\right)}{V_{\Sigma_u^-} - V_{\Sigma_g^+}} \quad (10)$$

$$\frac{\frac{c_F}{2m_Q} \int_{-\frac{T}{2}}^{\frac{T}{2}} dt \langle g \mathbf{B}(\frac{\mathbf{r}}{2}, t) g \mathbf{B}(\mathbf{0}, -\frac{T}{2}) - \hat{\mathbf{r}} \mathbf{g} \mathbf{B}(\frac{\mathbf{r}}{2}, t) \hat{\mathbf{r}} \mathbf{g} \mathbf{B}(\mathbf{0}, -\frac{T}{2}) \rangle_\square}{\langle 1 \rangle_\square^{1/2} \langle g \mathbf{B}(\mathbf{0}, \frac{T}{2}) g \mathbf{B}(\mathbf{0}, -\frac{T}{2}) - \hat{\mathbf{r}} \mathbf{g} \mathbf{B}(\mathbf{0}, \frac{T}{2}) \hat{\mathbf{r}} \mathbf{g} \mathbf{B}(\mathbf{0}, -\frac{T}{2}) \rangle_\square^{1/2}} = 2\sqrt{2} V_S^\Pi \frac{\sin\left(\frac{(V_{\Pi_u} - V_{\Sigma_g^+})T}{2}\right)}{V_{\Pi_u} - V_{\Sigma_g^+}}, \quad (11)$$

$\langle \dots \rangle_\square$ means expectation value in the Wilson loop, and c_F is an NRQCD hard matching coefficient. Notice that the Euclidean version of the objects on the lhs can be easily calculated on the lattice. At large T , V_S^Σ and V_S^Π can be then extracted by matching the data to the Euclidean version of the rhs, since $V_{\Sigma_g^+}$, $V_{\Sigma_u^-}$ and V_{Π_u} are known. In the following we are going to derive short and long distance constraints on these potentials using weak coupling pNRQCD [3, 15] and the EST respectively [12, 18].

At short distances the time evolution of a $Q\bar{Q}$ pair is described by pNRQCD at weak coupling [3, 15]. The leading spin-dependent term in the pNRQCD Lagrangian reads [19],

$$\mathcal{L}_{pNRQCD} = \frac{c_F}{2m_Q} \text{Tr} \left(O^\dagger(\mathbf{r}, \mathbf{R}, t) g \mathbf{B}(\mathbf{R}, t) \{ \sigma, S(\mathbf{r}, \mathbf{R}, t) \} \right) + \text{H.c.} \quad (12)$$

We use tr for trace over color indices and Tr for trace over both color and spin indices. Notice that the term above shows an \mathbf{r} -independent interaction between the single field and the operator $tr(O\mathbf{B})$, the short distance representation of \mathbf{H} , which implies that

$$V_S^\Sigma(r) = V_S^\Pi(r) = \pm \frac{c_F \lambda^2}{m_Q}, \quad (13)$$

where we have made the sign explicit, and $\lambda \sim \Lambda_{QCD}$ is a constant.

At long distances the energy spectrum of a static $Q\bar{Q}$ pair is well described by the EST [12, 18]. The mapping between operator insertions in the temporal Wilson lines of the Wilson loop and the corresponding operators in the EST was established in [20]. A mapping can also be established for operators in the space Wilson lines at $\pm T/2$ and states in the EST [21]. Hence, the expressions on the lhs of (10) can be calculated in the EST (as well as $V_{\Sigma_u^-}$, V_{Π_u} and $V_{\Sigma_g^+}$). We obtain,

$$V_S^\Sigma(r) = -\frac{\pi^2 \Lambda''' c_F}{m_Q k r^3}, \quad V_S^\Pi(r) = \frac{\pi^{3/2} \Lambda' c_F}{2m_Q \sqrt{k} r^2}. \quad (14)$$

The parameters $\Lambda' \sim \Lambda_{QCD}$ and $\Lambda''' \sim \Lambda_{QCD}$ also appear in the spin-orbit and tensor potentials of heavy quarkonium [20, 22], which have been calculated on the lattice. We obtain from fits to the data of ref. [23],

$$\kappa \sim 0.187 \text{GeV}^2, \quad \Lambda' \sim -118 \text{MeV}, \quad \Lambda''' \sim \pm 230 \text{MeV}, \quad (15)$$

where we have also displayed the value we use for the string tension κ .

For the actual mixing potentials, we use the simplest interpolation that allows for a sign flip between the short and long distance expressions without introducing any further scale. We have explored the following values for the only unknown parameter $\lambda = 100, 300, 600$ MeV, and all possible sign combinations. We present here our results for the phenomenologically relevant candidates to charmonium hybrids in table 3 ($S = 0$). For the 1^{++} and 1^{--} states a system of two and four coupled equations must be solved respectively.

We find that for the 1^{++} states, the mixing is small in all cases ($< 5\%$). For the 1^{--} states this is only so in the case that V_S^Σ flips sign and V_S^Π does not ($< 6\%$). In the cases that both V_S^Σ and V_S^Π flip sign or both keep the same sign, it may reach 15% for some states when $\lambda = 600$ MeV. The most dramatic case occurs when V_S^Σ keeps the same sign and V_S^Π flips it, the mixing ranges from 20% to 40% for all the states in table 3 when $\lambda = 600$ MeV. In this case the charmonium hybrid spectrum moves up about 10 – 20 MeV.

5 Conclusions

We have calculated the lower lying spectrum for charmonium and bottomonium hybrids at leading order in the BO approximation, including the $\Pi_u-\Sigma_u^-$ mixing terms, which were neglected in most of the earlier calculations [10, 24]. A detailed comparison with earlier work will be presented elsewhere [21]. Let us only mention here that the equations we get (4) are equivalent to those obtained in [25] (see also [26]), and our energy spectrum for H_1 and H_2 is at the lower end of the error bars in that reference whereas for H_3 it is in the middle. The basic difference with respect to ref. [25] is that we fix the arbitrary constant in the potential through the heavy quarkonium spectrum whereas in that reference it is fixed through the RS heavy quark masses [27]. We identify a number of possible heavy hybrid states in table 3, which would correspond to spin zero hybrids. The 1^{--} states would be the ground state and lower excitation of the $(s/d)_1$ (H_1) heavy hybrid states. However, spin symmetry violating decays have been observed for all of them, except for $X(4630)$.

Using weak coupling pNRQCD, we have also been able to estimate a limited number of decay widths to heavy quarkonium. In the way, we have learned that heavy hybrids with $J = L$ do not decay to lower lying heavy quarkonium states at leading order. This selects $X(4160)$ as a good candidate for the spin zero $1p_1$ (H_2) hybrid state.

We have pointed out that spin zero (one) heavy hybrids mix with spin one (zero) heavy quarkonium. We have calculated the asymptotic behavior at short and long distances for the mixing potentials, using weak coupling pNRQCD and the EST respectively. By building simple interpolations, we have also demonstrated that the mixing may be sizable (20% – 40%) for the 1^{--} charmonium hybrid states, which may explain the observed decays to spin one charmonium. As a consequence, $Y(4008)$, $Y(4360)$ and $Y(4660)$ remain as spin zero $(s/d)_1$ (H_1) heavy hybrid candidates.

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